Foundations of Garbled Circuits

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Outline

Paper overview

Definitions
- Garbling scheme
- Circuit
- Security

Security relations
- priv.sim $\Rightarrow$ priv.ind
- priv.ind $\wedge$ eff.inv $\Rightarrow$ priv.sim

Rest of the paper

Garbled Circuits

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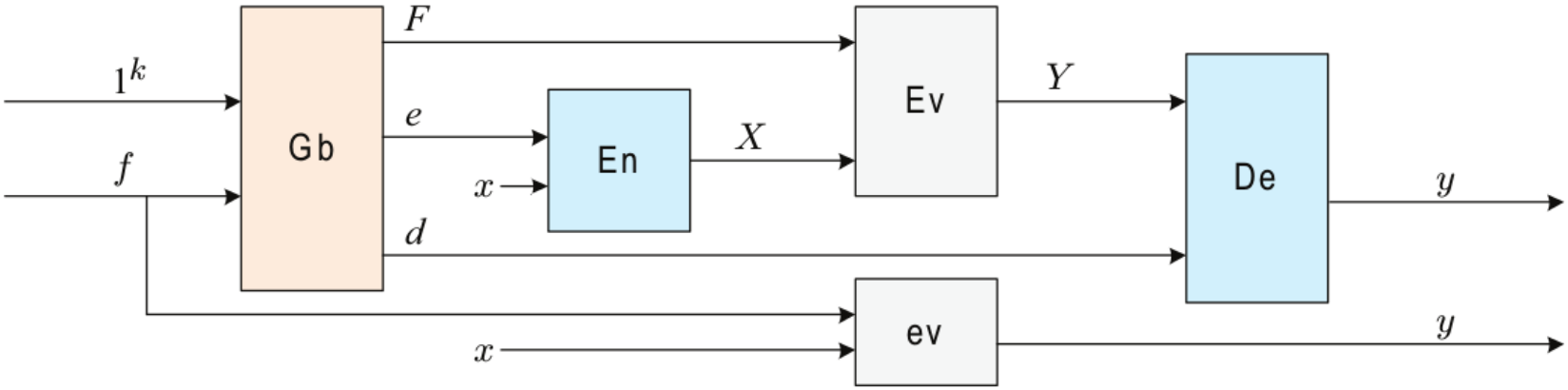
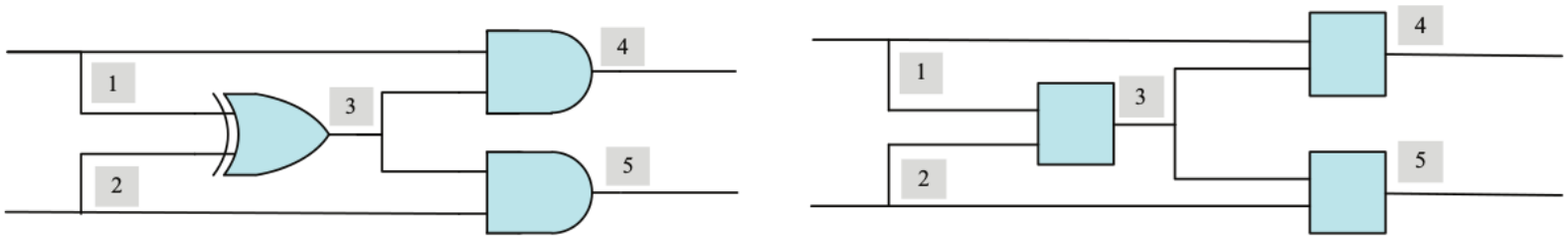
- Garbling as a goal, not a technique
- Garbling **scheme**
- Fit existing literature in the framework
- Examples: Garble1/Garble2
- Goal:
  - More efficient construction
  - More rigourous analyses
  - More modular design

Garbling scheme



- $\mathcal{G} = (\mathsf{Gb}, \mathsf{En}, \mathsf{De}, \mathsf{Ev}, \mathsf{ev})$
- Compute $F(X) = Y \sim f(x) = y$
- Gb: Garbler
- En, De: encrypter/decrypter
- Ev, ev: "interpreters"

Circuit



- $f = (n, m, q, A, B, G)$
- f is both an *encoding* of a function and the function itself
  - $\mathrm{ev}(f, x) = f(x)$

Security

- $\Phi(f)$: side-information on f
  - $\Phi_{size}(f) = (n, m, q)$
  - $\Phi_{topo}(f) = (n, m, q, A, B)$
  - $\Phi_{circ}(f) = (n, m, q, A, B, G) = f$
- Privacy
  - (F,X,d) reveals nothing beyond $\Phi(f)$ and y
- Obliviousness
  - (F,X) reveals nothing beyond $\Phi(f)$
- Authenticity
  - Given F, X, adversary is unable to produce Y^*, s.t. $d(Y^*) \neq \perp$

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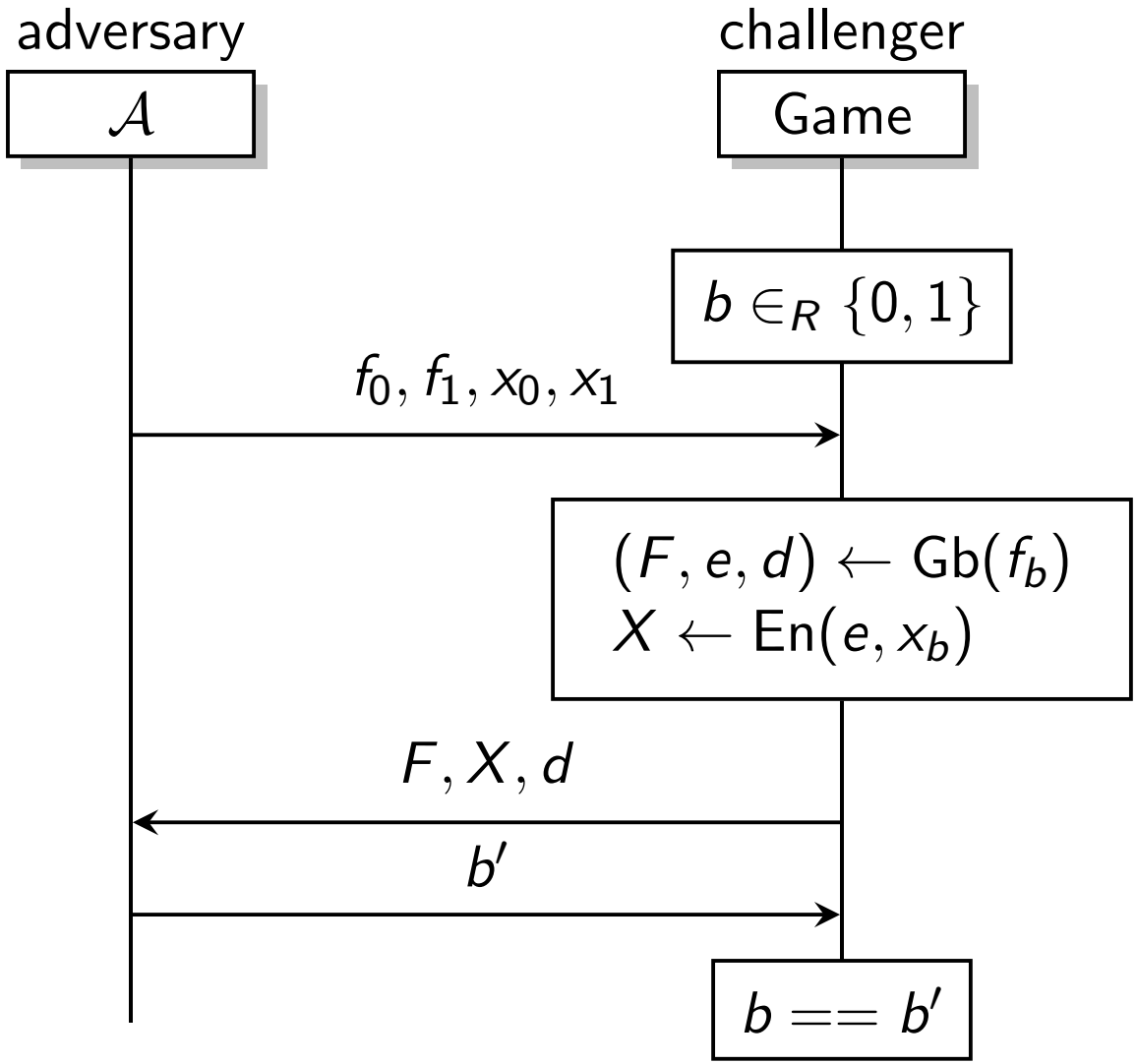
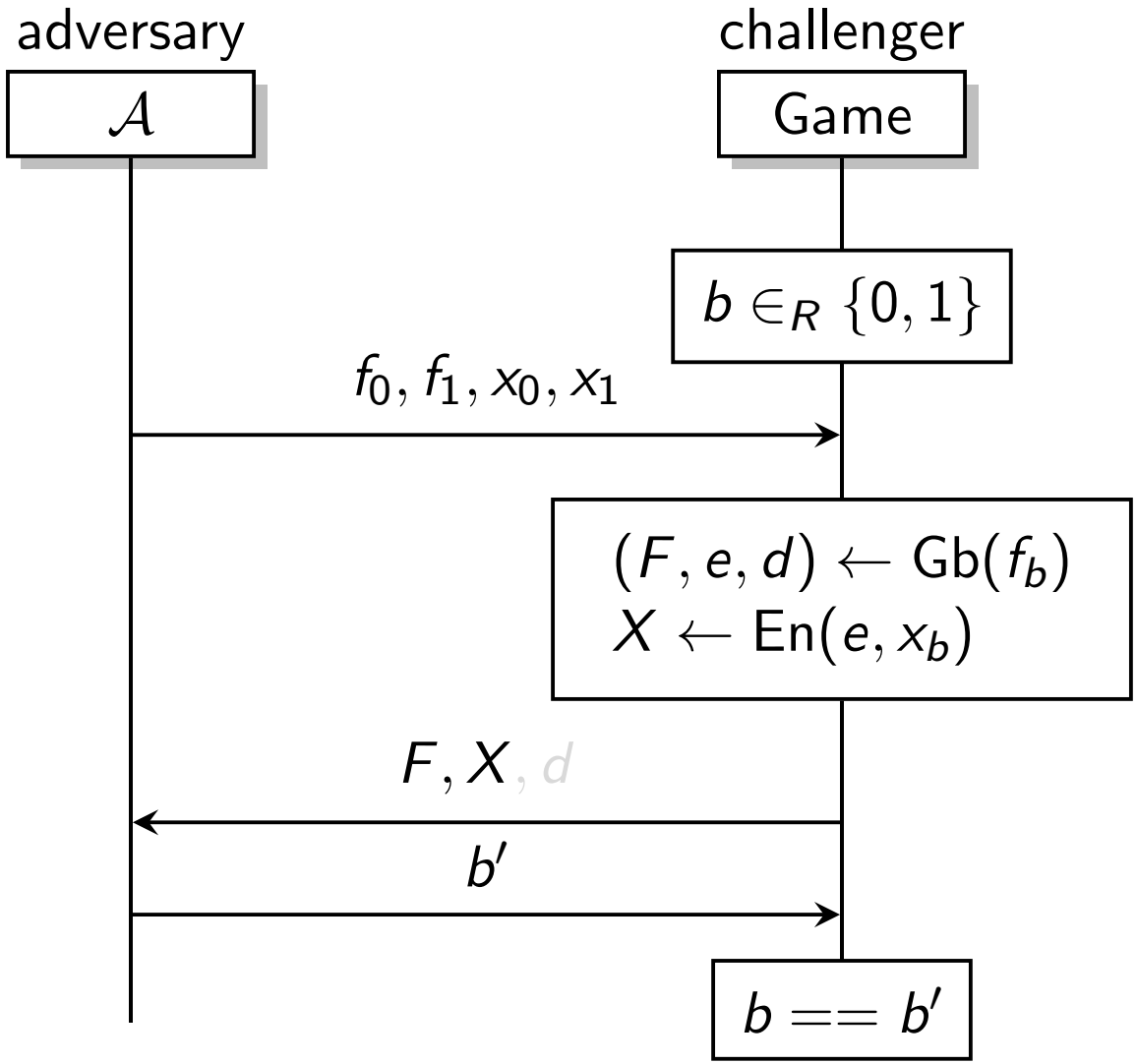
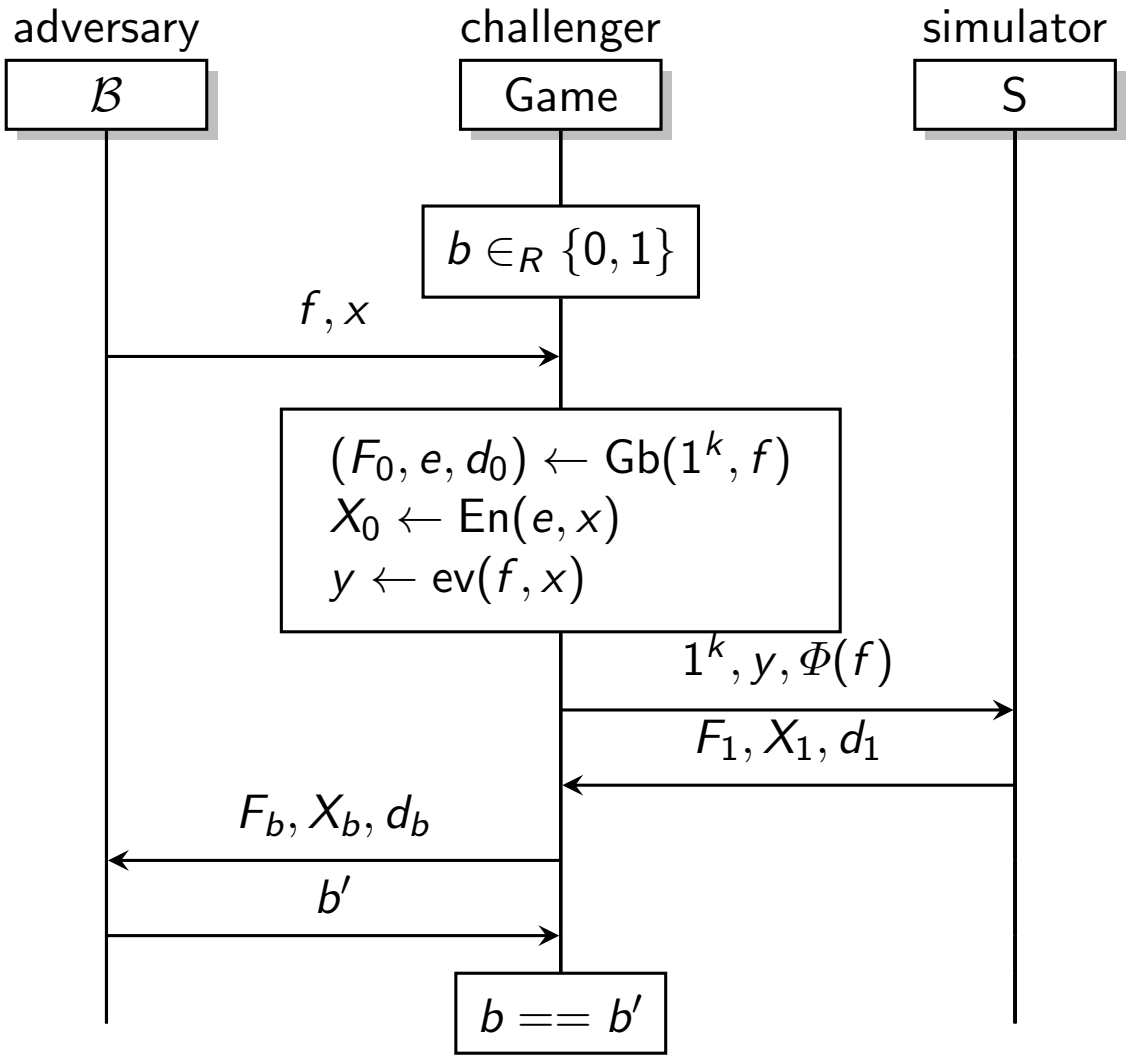
Indistinguishability (privacy)



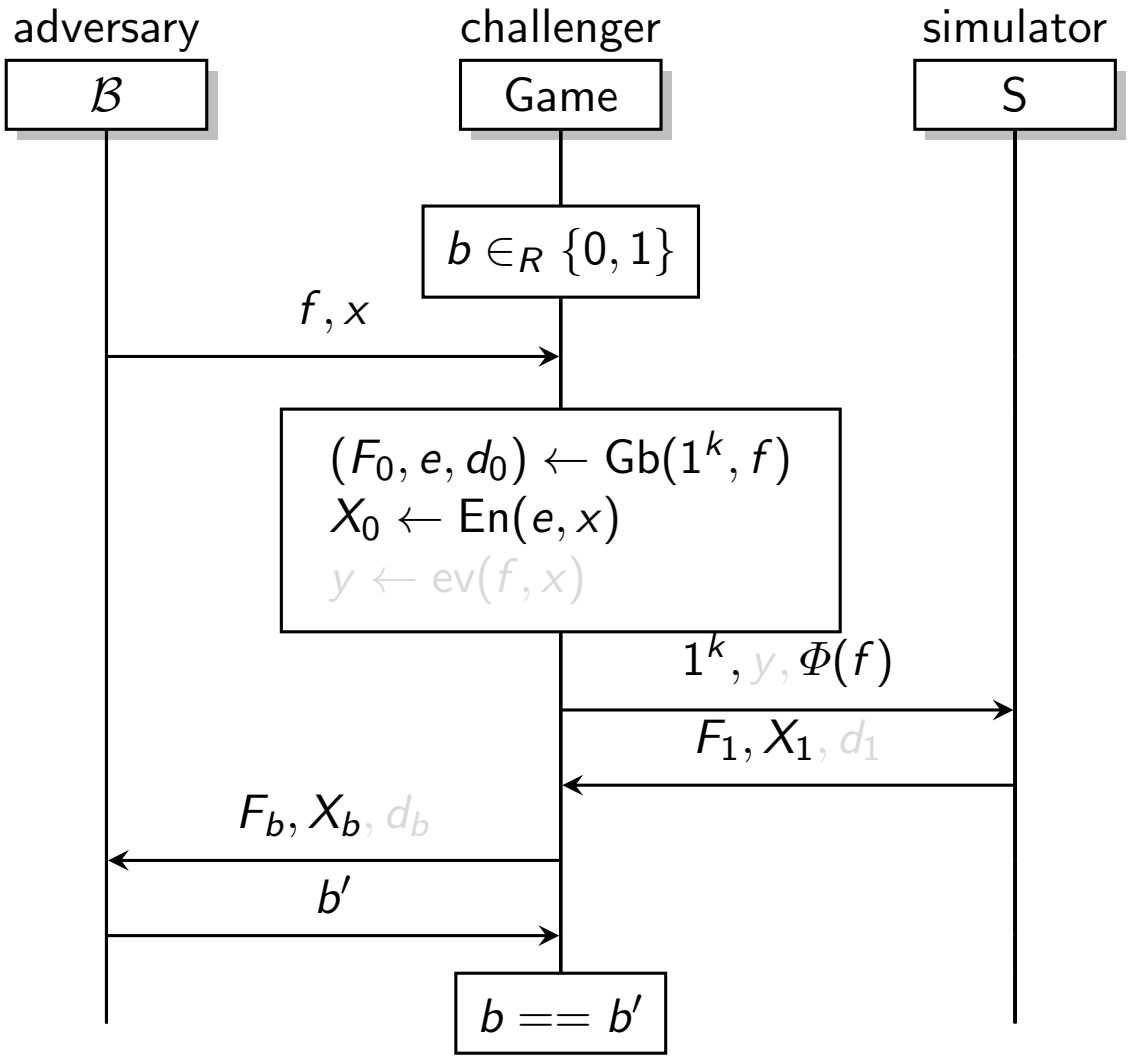
Indistinguishability (obliviousness)



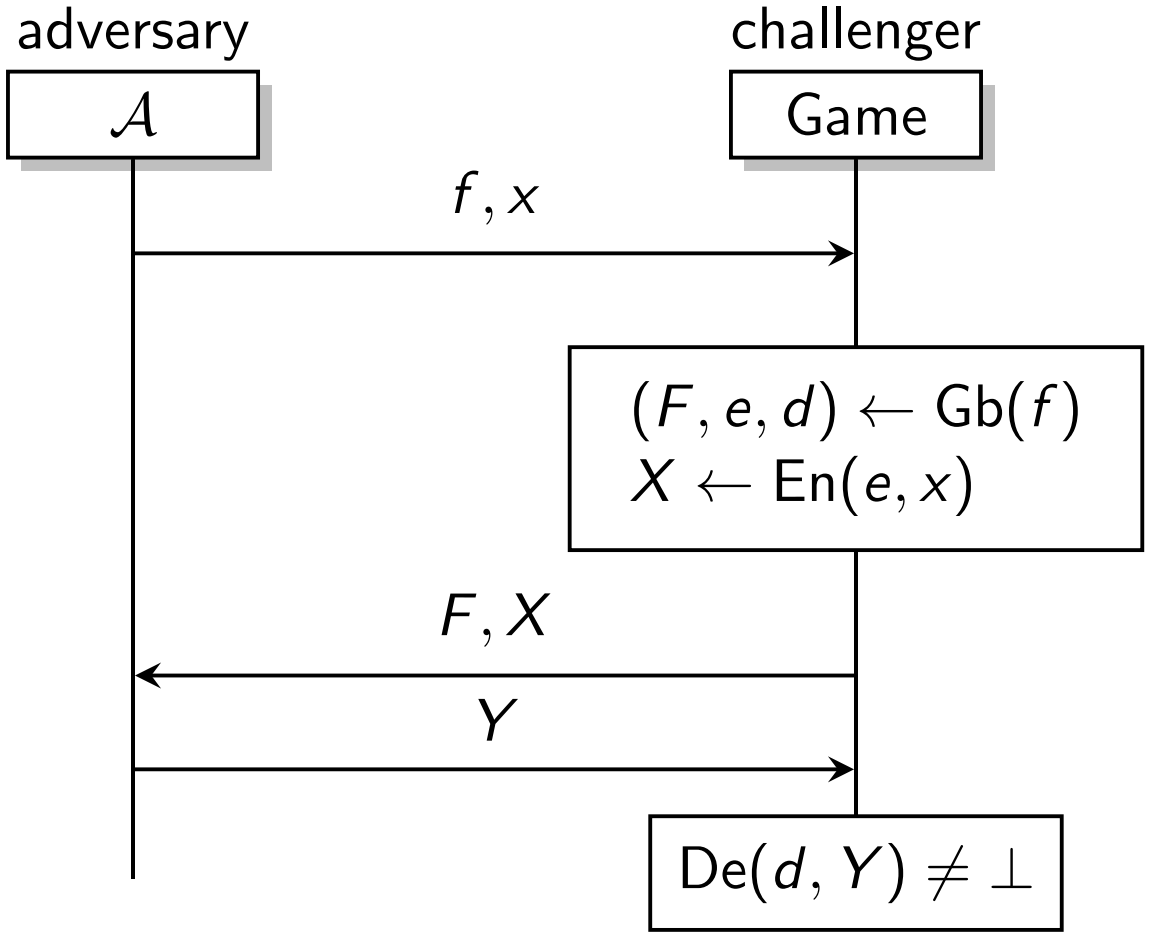
Simulation (privacy)



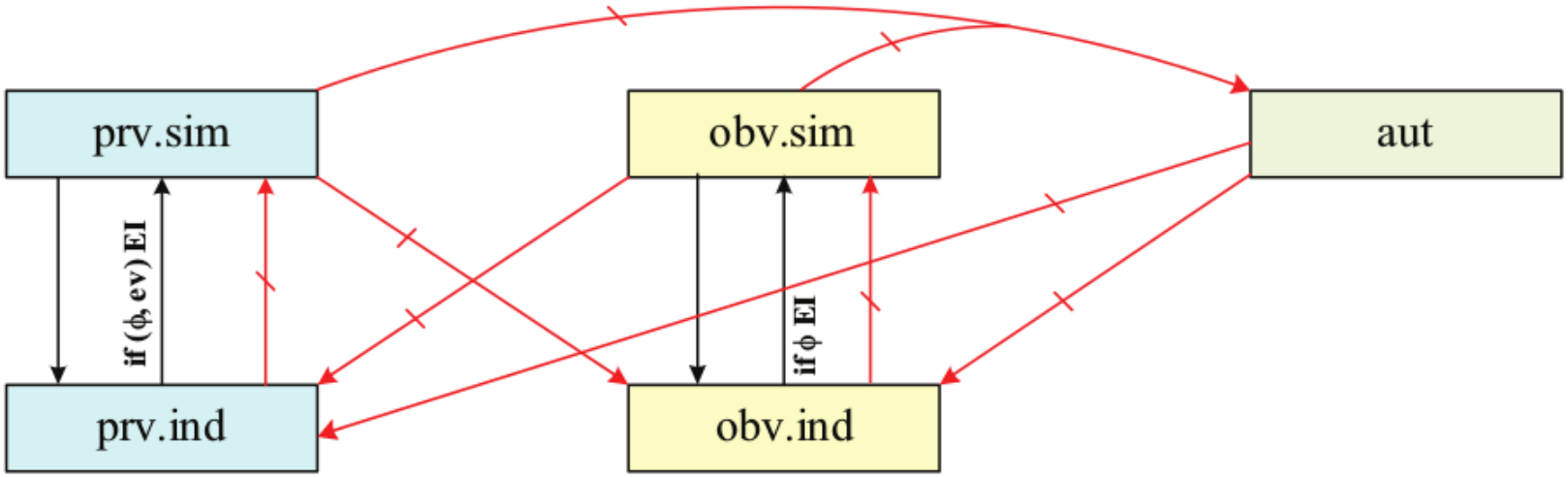
Simulation (obliviousness)



Authenticity

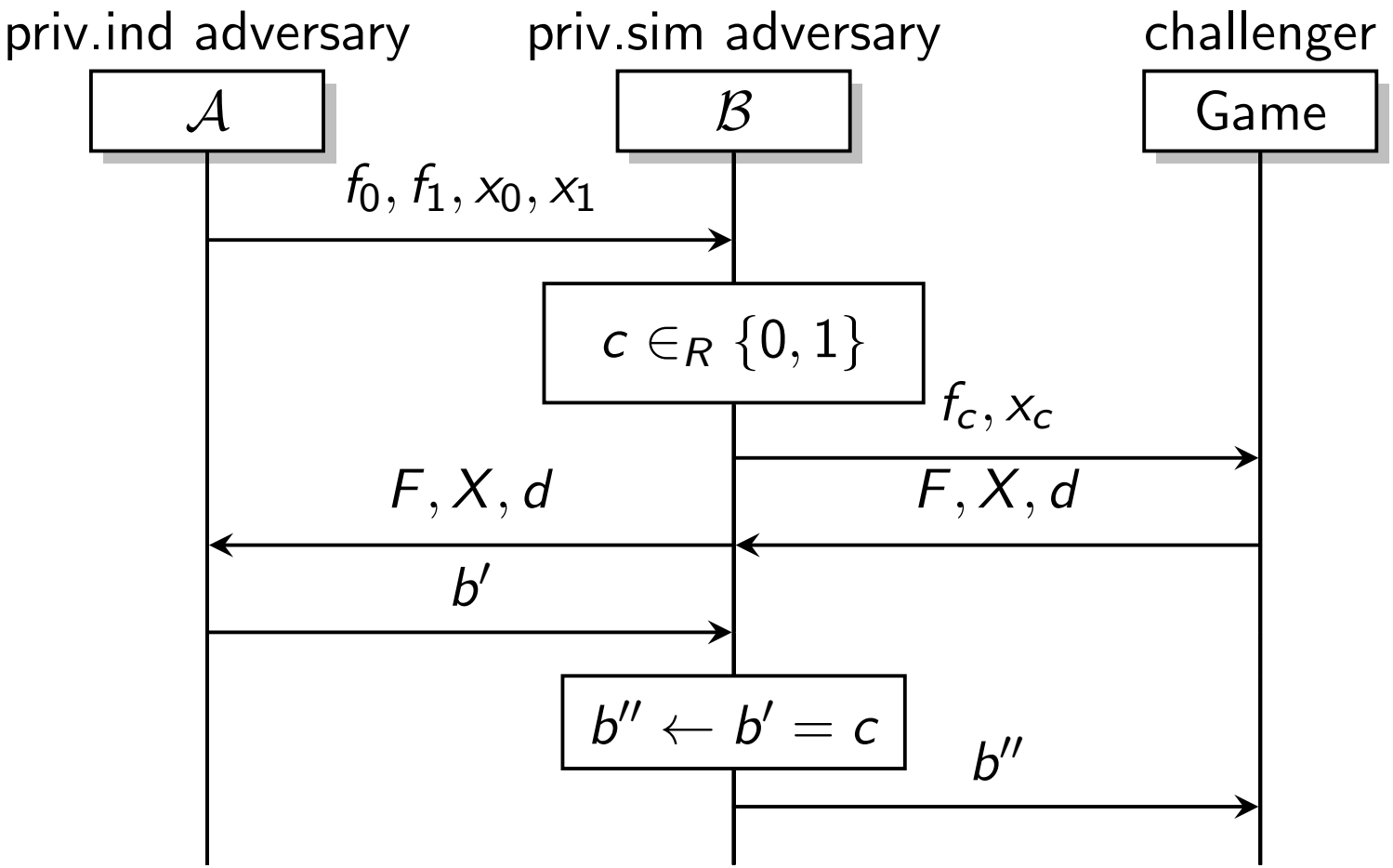


Security relations



- GS(priv.sim, Φ) is the set of all garbling schemes that are privacy simulation secure over Φ
- similar for priv.ind, obv.sim, obv.ind
- similar for aut, but without Φ

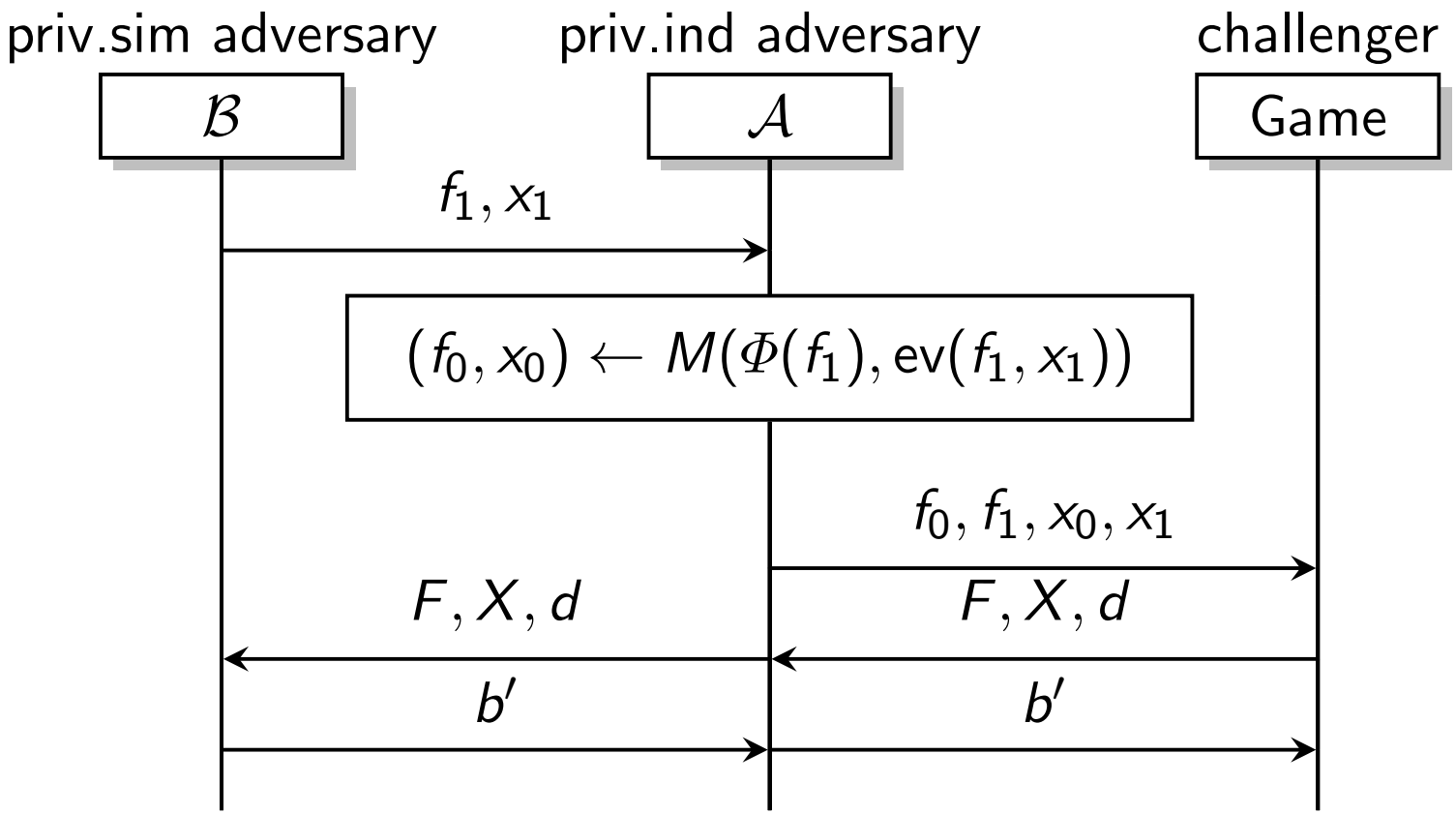
priv.sim $\Rightarrow$ priv.ind



Efficient invertibility

- M is a Φ-inverter if
  - $M(\phi)$ returns f s.t. $\Phi(f) = \phi$
- M is a (Φ, ev)-inverter if
  - $M(\phi, y)$ returns (f, x) s.t. $\Phi(f) = \phi$ and $\mathsf{ev}(f, x) = y$
- Efficient inverters do it in polynomial time

priv.ind $\wedge$ eff.inv $\Rightarrow$ priv.sim



Rest of the paper

- Proofs for the other drawn security relations
- Garble1
  - Definition
  - Dual-key ciphers
  - Proof of security (priv.ind over Φ_{topo})
- Garble2
  - Definition
  - Proof of security
    - priv.ind over $\Phi_{topo}(\Rightarrow priv.sim)$
    - obv.ind over $\Phi_{topo}(\Rightarrow obv.sim)$
    - aut
- Casting existing schemes to the GS framework
  - Secure function evaluation (SFE)
  - Private function evaluation (PFE)

Thank you

Any questions?